$D \times f_{ib} \neq 0$

No.
Date. 21 · 6 · 2017

<u>continuous case:</u>

$$f_1 - f_2 = \int_j f \cos(\theta) \left[\Phi(r_{1j}) - \Phi(r_{2j}) \right] ds$$

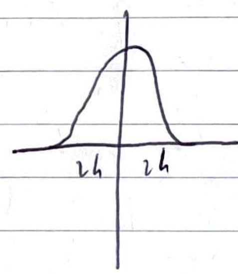
$$f_b - f_a = \int_j f \sin(\theta) \left[\Phi(r_{bj}) - \Phi(r_{aj}) \right] ds$$

ϕ = even function, $\phi \sim 1, x^2, x^4, x^6, \ldots$

An example is Chebychev 1st order function,

$$\phi(x) = \alpha(1 + \cos \beta x) \qquad |x| \leq 2h$$

$$= \alpha\left(1 + \cos \frac{\pi x}{2h}\right)$$



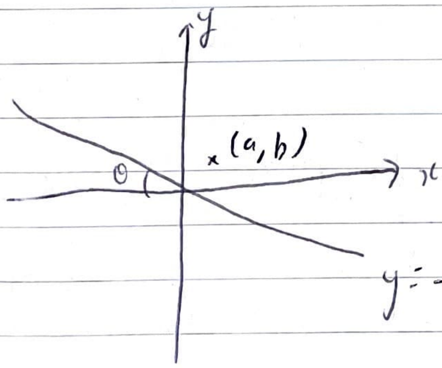
$\int \phi \, dx = 1$

$$\int_{-2h}^{2h} \phi \, dx = 1 = 4h \cdot \alpha + \left[\frac{2h}{\pi} \sin \frac{\pi x}{2h} \right]_{-2h}^{2h}$$

$$\alpha = \frac{1}{4h}$$

$$\therefore \phi(x) = \frac{1}{4h}\left(1 + \cos \frac{\pi x}{2h}\right)$$

$$\frac{d\phi}{dx} = -\frac{\pi}{8h^2} \sin \frac{\pi x}{2h}$$



$y = -x \tan\theta$, $0 \leq \theta \leq \frac{\pi}{2}$

No.
Date. 5 · 7 · 2017

$D_x f_{ib} \neq 0$

Integration limits:

$$D_x f_{ib} = \frac{f_y(a+\frac{1}{2}h, b) - f_y(a-\frac{1}{2}h, b)}{h} - \frac{f_x(a, b+\frac{1}{2}h) - f_x(a, b-\frac{1}{2}h)}{h}$$

$$f_y(a+\tfrac{1}{2}h, b) = \int_{\alpha_1}^{\beta_1} f\cos\theta \cdot \phi(x-a-\tfrac{1}{2}h)\phi(y-b)\,dx/|\cos\theta|$$

$$f_y(a-\tfrac{1}{2}h, b) = \int_{\alpha_2}^{\beta_2} f\cos\theta \cdot \phi(x-a+\tfrac{1}{2}h)\phi(y-b)\,dx/|\cos\theta|$$

$$f_x(a, b+\tfrac{1}{2}h) = \int_{\delta_1}^{\gamma_1} f\sin\theta \cdot \phi(x-a)\phi(y-b-\tfrac{1}{2}h)\,dx/|\cos\theta|$$

$$f_x(a, b-\tfrac{1}{2}h) = \int_{\delta_2}^{\gamma_2} f\sin\theta \cdot \phi(x-a)\phi(y-b+\tfrac{1}{2}h)\,dx/|\cos\theta|$$

$\alpha_1, \beta_1, \alpha_2, \beta_2, \delta_1, \gamma_1, \delta_2, \gamma_2$

$y = -x\tan\theta$

for $(a+\frac{1}{2}h, b)$, if $x = a-\frac{3}{2}h$ if $x = a+\frac{5}{2}h$

$b+2h = y = -(a-\frac{3}{2}h)\tan\theta$ $b-2h = y = -(a+\frac{5}{2}h)\tan\theta$

$\tan\theta = -\frac{b+2h}{a-\frac{3}{2}h}$ $\tan\theta = -\frac{b-2h}{a+\frac{5}{2}h}$

if $\theta \le \tan^{-1}\left[-\frac{b+2h}{a-\frac{3}{2}h}\right]$ $\alpha_1 = a-\frac{3}{2}h$, else $\alpha_1 = -\frac{b+2h}{\tan\theta}$

if $\theta \le \tan^{-1}\left[-\frac{b-2h}{a+\frac{5}{2}h}\right]$ $\beta_1 = a+\frac{5}{2}h$, else $\beta_1 = -\frac{b-2h}{\tan\theta}$

$D_x f_{i,k} \neq 0$

No.
Date. 5 · 7 · 2017

for $(a-\frac{1}{2}h, b)$, if $x = a-\frac{5}{2}h$, if $x = a+\frac{3}{2}h$

$b+2h = y = -(a-\frac{5}{2}h)\tan\theta$ $\quad$ $b-2h = y = -(a+\frac{3}{2}h)\tan\theta$

$\tan\theta = -\dfrac{b+2h}{a-\frac{5}{2}h}$ $\quad$ $\tan\theta = -\dfrac{b-2h}{a+\frac{3}{2}h}$

if $\theta \le \tan^{-1}\left[-\dfrac{b+2h}{a-\frac{5}{2}h}\right]$, $\alpha_2 = a-\frac{5}{2}h$, else $\alpha_2 = -\dfrac{b+2h}{\tan\theta}$

if $\theta \le \tan^{-1}\left[-\dfrac{b-2h}{a+\frac{3}{2}h}\right]$, $\beta_2 = a+\frac{3}{2}h$, else $\beta_2 = -\dfrac{b-2h}{\tan\theta}$

for $(a, b+\frac{1}{2}h)$, if $x = a-2h$, if $x = a+2h$

$\tan\theta = -\dfrac{b+\frac{5}{2}h}{a-2h}$ $\quad$ $\tan\theta = -\dfrac{b-\frac{3}{2}h}{a+2h}$

if $\theta \le \tan^{-1}\left[-\dfrac{b+\frac{5}{2}h}{a-2h}\right]$, $\delta_1 = a-2h$, else $\delta_1 = -\dfrac{b+\frac{5}{2}h}{\tan\theta}$

if $\theta \le \tan^{-1}\left[-\dfrac{b-2h}{a+\frac{3}{2}h}\right]$, $\cancel{\delta_1}\ \gamma_1 = a+2h$, else $\gamma_1 = -\dfrac{b-\frac{3}{2}h}{\tan\theta}$

for $(a, b-\frac{1}{2}h)$, if $x = a-2h$, if $x = a+2h$

$\tan\theta = -\dfrac{b+\frac{3}{2}h}{a-2h}$ $\quad$ $\tan\theta = -\dfrac{b-\frac{5}{2}h}{a+2h}$

if $\theta \le \tan^{-1}\left[-\dfrac{b+\frac{3}{2}h}{a-2h}\right]$, $\delta_2 = a-2h$, else $\delta_2 = -\dfrac{b+\frac{3}{2}h}{\tan\theta}$

if $\theta \le \tan^{-1}\left[-\dfrac{b-\frac{5}{2}h}{a+2h}\right]$, $\gamma_2 = a+2h$, else $\gamma_2 = -\dfrac{b-\frac{5}{2}h}{\tan\theta}$

No.
Date. 5 · 7 · 2017

$D_x f_{ij} \neq 0$

if $a = b = 0$, $D_x f_{ib} = 0$ trivially, because :

$\alpha_1 = -\beta_2$, $\beta_1 = -\alpha_2$, consider even polynomial series

$(1, x^2, x^4, x^6, \ldots)$
$(g_0, g_1, g_2, g_3, \ldots)$

$$\frac{f_y(a+\frac{1}{2}h, b) - f_y(a-\frac{1}{2}h, b)}{h}$$

$$= \int_{\alpha_1}^{\beta_1} (g_0 + g_1 x^2 + g_2 x^4 + \ldots)(g_0' + g_1'(x-\tfrac{1}{2}h)^2 + g_2'(x-\tfrac{1}{2}h)^4 + \ldots)\,dx$$

$$- \int_{-\beta_1}^{-\alpha_1} (g_0 + g_1 x^2 + g_2 x^4 + \ldots)(g_0' + g_1'(x+\tfrac{1}{2}h)^2 + g_2'(x+\tfrac{1}{2}h)^4 + \ldots)\,dx$$

all the common constats are absorbed into $g_0', g_1', \ldots$

each integrand has the form of $x^{2m}(x-\frac{1}{2}h)^{2n}$ &
$x^{2m}(x+\frac{1}{2}h)^{2n}$

So let's investigate term by term, eventually they will cancel out each other.

$$\int_{\alpha_1}^{\beta_1} x^{2m}(x-\tfrac{1}{2}h)^{2n}\,dx - \int_{-\beta_1}^{-\alpha_1} x^{2m}(x+\tfrac{1}{2}h)^{2n}\,dx$$

$D \times f_{ib} \neq 0$

$$= \int_{\alpha_1}^{\beta_1} x^{2m} \left(x^{2n} + {}_{2n}C_1 x^{2n-1} \left(-\tfrac{1}{2}h\right) + {}_{2n}C_2 x^{2n-2} \left(-\tfrac{1}{2}h\right)^2 + \dots \right) dx$$
$$- \int_{-\beta_1}^{-\alpha_1} x^{2m} \left(x^{2n} + {}_{2n}C_1 x^{2n-1} \cdot \tfrac{1}{2}h + {}_{2n}C_2 x^{2n-2} \left(\tfrac{1}{2}h\right)^2 + \dots \right) dx$$

for even degree term,

$$\int_{\alpha_1}^{\beta_1} x^{2k}\,dx - \int_{-\beta_1}^{-\alpha_1} x^{2k}\,dx = \frac{1}{2k+1}\left[x^{2k+1}\right]_{\alpha_1}^{\beta_1} - \left[x^{2k+1}\right]_{-\beta_1}^{-\alpha_1}\frac{1}{2k+1}$$
$$= \frac{1}{2k+1}\left[\beta_1^{2k+1} - \alpha_1^{2k+1} + \alpha_1^{2k+1} - \beta_1^{2k+1}\right] = 0$$

for odd degree term,

$$\int_{\alpha}^{\beta_1} x^{2k+1}\,dx - \int_{-\beta_1}^{-\alpha_1} -x^{2k+1}\,dx = \frac{1}{2k+2}\left[x^{2k+2}\right]_{\alpha}^{\beta_1} + \frac{1}{2k+2}\left[x^{2k+2}\right]_{-\beta_1}^{-\alpha_1}$$
$$= \frac{1}{2k+2}\left[\beta_1^{2k+2} - \alpha_1^{2k+2} + \alpha_1^{2k+2} - \beta_1^{2k+2}\right] = 0$$

similarly for $\dfrac{f_x(a, b+\frac{1}{2}h) - f_x(a, b-\frac{1}{2}h)}{h}$

$\therefore D \times f_{ib} = 0$ when $h \to 0$

---

if $a \neq 0$, $b \neq 0$, $D \times f_{ib} \neq 0$

there seems to be no general way to prove it,

but for $\phi(x) = \frac{1}{4h}\left(1 + \cos\frac{\pi x}{2h}\right)$

No.

Date. 5 · 7 · 2017

Dx Fib fo

for small angle θ, $\alpha_1 = a - \frac{3}{2}h$, $\beta_1 = a + \frac{5}{2}h$

$\alpha_2 = a - \frac{5}{2}h$, $\beta_2 = a + \frac{3}{2}h$

$\delta_1 = a - 2h$, $\gamma_1 = a + 2h$

$\delta_2 = a - 2h$, $\gamma_2 = a + 2h$

$$\int_{\alpha_1}^{\beta_1} f \phi(x - a - \tfrac{1}{2}h)\phi(y-b)\,dx$$

$$= f \int_{\alpha_1}^{\beta_1} \frac{1}{(4h)^2}\left(1 + \cos\frac{x-a-\frac{1}{2}h}{2h}\pi\right)\left(1 + \cos\frac{-x\tan\theta - b}{2h}\pi\right)dx$$

$$= f \int_{\alpha_1}^{\beta_1} \frac{1}{(4h)^2}\left(1 + \cos\frac{x-a-\frac{1}{2}h}{2h}\pi + \cos\frac{x\tan\theta + b}{2h}\pi + \left(\cos\frac{x-a-\frac{1}{2}h}{2h}\pi\right)\left(\cos\frac{x\tan\theta+b}{2h}\pi\right)\right)dx$$

$$= \frac{f}{16h^2}\int_{\alpha_1}^{\beta_1}\left(1 + \cos\frac{x-a-\frac{1}{2}h}{2h}\pi + \cos\frac{x\tan\theta+b}{2h}\pi + \frac{1}{2}\cos\frac{(1+\tan\theta)x - a + b - \frac{1}{2}h}{2h}\pi + \frac{1}{2}\cos\frac{(1-\tan\theta)x - a - b - \frac{1}{2}h}{2h}\pi\right)dx$$

$$= \frac{f}{16h^2}(\beta_1 - \alpha_1) + \frac{f}{8h\pi}\left[\sin\frac{x-a-\frac{1}{2}h}{2h}\pi\right]_{\alpha_1}^{\beta_1} + \frac{f}{8h\tan\theta\cdot\pi}\left[\sin\frac{x\tan\theta+b}{2h}\pi\right]_{\alpha_1}^{\beta_1}$$

$$+ \frac{f}{16h\pi(1+\tan\theta)}\left[\sin\frac{(1+\tan\theta)x - a + b - \frac{1}{2}h}{2h}\pi\right]_{\alpha_1}^{\beta_1}$$

$$+ \frac{f}{16h\pi(1-\tan\theta)}\left[\sin\frac{(1-\tan\theta)x - a - b - \frac{1}{2}h}{2h}\pi\right]_{\alpha_1}^{\beta_1}$$

let $a = a'h$, $b = b'h$, $-2 \le a' \le 2$, $-2 \le b' \le 2$

$D \times f_{ib} \neq 0$

No.
Date. 5 · 7 · 2017

$$\therefore = \frac{f}{4h} + \frac{f}{8h\pi}\left[\sin\pi - \sin(-\pi)\right] + \frac{f}{8h\pi\tan\theta}\left[\sin\frac{(a'+\frac{5}{2})\tan\theta+b'}{2}\pi\right.$$
$$\left. - \sin\frac{(a'-\frac{3}{2})\tan\theta+b'}{2}\pi\right] + \frac{f}{16h\pi(\tan\theta+1)}\left[\sin\frac{(a'+\frac{5}{2})\tan\theta+b'+2}{2}\pi\right.$$
$$\left. - \sin\frac{(a'-\frac{3}{2})\tan\theta+b'-2}{2}\pi\right] + \frac{f}{16h\pi(1-\tan\theta)}\left[\sin\frac{-(a'+\frac{5}{2})\tan\theta-b'+2}{2}\pi\right.$$
$$\left. + \sin\frac{(a'-\frac{3}{2})\tan\theta+b'+2}{2}\pi\right]$$

similarly,

$$\int_{\alpha_2}^{\beta_2} f\phi(x-a+\tfrac{1}{2}h)\phi(y-b)\,dx$$

$$= \frac{f}{16h^2}\int_{\alpha_2}^{\beta_2}\left(1+\cos\frac{x-a+\frac{1}{2}h}{2h}\pi + \cos\frac{x\tan\theta+b}{2h}\pi + \frac{1}{2}\cos\frac{(1+\tan\theta)x-a+b+\frac{1}{2}h}{2h}\pi\right.$$
$$\left. + \frac{1}{2}\cos\frac{(1-\tan\theta)x-a-b+\frac{1}{2}h}{2h}\pi\right)dx$$

$$= \frac{f}{16h^2}\left[\beta_2-\alpha_2\right] + \frac{f}{8h\pi}\left[\sin\frac{x-a+\frac{1}{2}h}{2h}\pi\right]_{\alpha_2}^{\beta_2} + \frac{f}{8h\pi\tan\theta}\left[\sin\frac{x\tan\theta+b}{2h}\pi\right]_{\alpha_2}^{\beta_2}$$
$$+ \frac{f}{16h\pi(1+\tan\theta)}\left[\sin\frac{(1+\tan\theta)x-a+b+\frac{1}{2}h}{2h}\pi\right]_{\alpha_2}^{\beta_2}$$
$$+ \frac{f}{16h\pi(1-\tan\theta)}\left[\sin\frac{(1-\tan\theta)x-a-b+\frac{1}{2}h}{2h}\pi\right]_{\alpha_2}^{\beta_2}$$

No.

Date. 5 · 7 · 2017

$D \times f_{ib} \neq 0$

$$= \frac{f}{4h} + \frac{f}{8h\pi}\left[\sin\pi - \sin(-\pi)\right] + \frac{f}{8h\pi\tan\theta}\left[\sin\frac{(a'+\frac{3}{2})\tan\theta+b'}{2}\pi\right.$$

$$\left.- \sin\frac{(a'-\frac{5}{2})\tan\theta+b'}{2}\pi\right] + \frac{f}{16h\pi(1+\tan\theta)}\left[\sin\frac{(a'+\frac{3}{2})\tan\theta+b'+2}{2}\pi\right.$$

$$\left.- \sin\frac{(a'-\frac{5}{2})\tan\theta+b'-2}{2}\pi\right] + \frac{f}{16h\pi(1-\tan\theta)}\left[\sin\frac{-(a'+\frac{3}{2})\tan\theta-b'+2}{2}\pi\right.$$

$$\left.+ \sin\frac{(a'-\frac{5}{2})\tan\theta+b'+2}{2}\pi\right]$$

$$\therefore \frac{f_y(a+\frac{1}{2}h, b) - f_y(a-\frac{1}{2}h, b)}{h}$$

$$= \frac{f}{8h^2\pi\tan\theta}\left[-2\sin\frac{a'}{2}\pi\cos\frac{\frac{3}{2}\tan\theta+b'}{2}\pi + 2\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{5\tan\theta}{4}\pi\right]$$

$$+ \frac{f}{16h^2\pi(1+\tan\theta)}\left[2\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{\frac{5}{2}\tan\theta+2}{2}\pi\right.$$

$$\left.- 2\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{\frac{3}{2}\tan\theta+2}{2}\pi\right] + \frac{f}{16h^2\pi(1-\tan\theta)}\left[\sin\frac{(a'+\frac{5}{2})\tan\theta+b'}{2}\pi\right.$$

$$\left.- \sin\frac{(a'-\frac{3}{2})\tan\theta+b'}{2}\pi - \sin\frac{(a'+\frac{3}{2})\tan\theta+b'}{2}\pi + \sin\frac{(a'-\frac{5}{2})\tan\theta+b'}{2}\pi\right]$$

$$= \frac{f}{4h^2\pi\tan\theta}\left[-\sin\frac{a'}{2}\pi\cos\frac{\frac{3}{2}\tan\theta+b'}{2}\pi + \sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{5\tan\theta}{4}\pi\right]$$

$$+ \frac{f}{8h^2\pi(1+\tan\theta)}\left[\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{3\tan\theta}{4}\pi - \sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{5\tan\theta}{4}\pi\right]$$

$$+ \frac{f}{8h^2\pi(1-\tan\theta)}\left[\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{5\tan\theta}{4}\pi - \sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{3\tan\theta}{4}\pi\right]$$

$D \times f_{ib} \neq 0$

No.

Date. 5 · 7 · 2017

$$= \frac{f}{4h^2\pi\tan\theta}\left[\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{5\tan\theta}{4}\pi - \sin\frac{a'}{2}\pi\cos\frac{\frac{3}{2}\tan\theta+b'}{2}\pi\right]$$

$$+ \frac{f}{8h^2\pi}\sin\frac{a'\tan\theta+b'}{2}\pi\left[\frac{1}{1+\tan\theta}-\frac{1}{1-\tan\theta}\right]\left(\cos\frac{3\tan\theta}{4}\pi - \cos\frac{5\tan\theta}{4}\pi\right).$$

$$\int_{\delta_1}^{\gamma_1} f'\phi(x-a)\phi\left(y-b-\tfrac{1}{2}h\right)dx\cdot\tan\theta \qquad f' = \frac{\cos\theta}{|\cos\theta|}$$

$$= \int_{\delta_1}^{\gamma_1}\frac{f}{16h^2}\left(1+\cos\frac{x-a}{2h}\pi\right)\left(1+\cos\frac{-x\tan\theta-b-\frac{1}{2}h}{2h}\pi\right)dx\cdot\tan\theta$$

$$= \frac{f\tan\theta}{16h^2}\int_{\delta_1}^{\gamma_1}\left(1+\cos\frac{x-a}{2h}\pi+\cos\frac{x\tan\theta+b+\frac{1}{2}h}{2h}\pi\right.$$

$$\left.+\frac{1}{2}\cos\frac{(1+\tan\theta)x-a+b+\frac{1}{2}h}{2h}\pi+\frac{1}{2}\cos\frac{(1-\tan\theta)x-a-b-\frac{1}{2}h}{2h}\pi\right)dx$$

$$= \frac{f\tan\theta}{16h^2}(\gamma_1-\delta_1)+\frac{f\tan\theta}{8h\pi}\left[\sin\frac{x-a}{2h}\pi\right]_{\delta_1}^{\gamma_1}+\frac{f}{8h\pi}\left[\sin\frac{x\tan\theta+b+\frac{1}{2}h}{2h}\pi\right]_{\delta_1}^{\gamma_1}$$

$$+\frac{f\tan\theta}{16h\pi(1+\tan\theta)}\left[\sin\frac{(1+\tan\theta)x-a+b+\frac{1}{2}h}{2h}\pi\right]_{\delta_1}^{\gamma_1}$$

$$+\frac{f\tan\theta}{16h\pi(1-\tan\theta)}\left[\sin\frac{(1-\tan\theta)x-a-b-\frac{1}{2}h}{2h}\pi\right]_{\delta_1}^{\gamma_1}$$

$$\int_{\delta_2}^{\gamma_2} f\phi(x-a)\phi\left(y-b+\tfrac{1}{2}h\right)dx\cdot\tan\theta$$

$$= \int_{\delta_2}^{\gamma_2}\frac{f}{16h^2}\left(1+\cos\frac{x-a}{2h}\pi\right)\left(1+\cos\frac{-x\tan\theta-b+\frac{1}{2}h}{2h}\pi\right)dx\cdot\tan\theta$$

No.

Date. 6 · 7 · 2017

$D \times f_{1,1} \neq 0$

$$= \frac{f\tan\theta}{16h^2}\int_{\delta_2}^{\gamma_2}\left(1+\cos\frac{x-a}{2h}\pi+\cos\frac{-x\tan\theta-b+\frac{5}{2}h}{2h}\pi\right.$$
$$\left.+\frac{1}{2}\cos\frac{(1-\tan\theta)x-a-b+\frac{5}{2}h}{2h}\pi+\frac{1}{2}\cos\frac{(1+\tan\theta)x-a+b-\frac{5}{2}h}{2h}\right)dx$$

$$= \frac{f\tan\theta}{16h^2}(\gamma_2-\delta_2)+\frac{f\tan\theta}{8h\pi}\left[\sin\frac{x-a}{2h}\pi\right]_{\delta_2}^{\gamma_2}+\frac{f}{8h\pi}\left[\sin\frac{x\tan\theta+b-\frac{5}{2}h}{2h}\pi\right]_{\delta_2}^{\gamma_2}$$
$$+\frac{f\tan\theta}{16h\pi(1-\tan\theta)}\left[\sin\frac{(1-\tan\theta)x-a-b+\frac{5}{2}h}{2h}\pi\right]_{\delta_2}^{\gamma_2}$$
$$+\frac{f\tan\theta}{16h\pi(1+\tan\theta)}\left[\sin\frac{(1+\tan\theta)x-a+b-\frac{5}{2}h}{2h}\pi\right]_{\delta_2}^{\gamma_2}$$

$$= \frac{f\tan\theta}{4h}+\frac{f\tan\theta}{8h\pi}\left[\sin\pi-\sin(-\pi)\right]+\frac{f}{8h\pi}\left[\sin\frac{(a'+2)\tan\theta+b'-\frac{5}{2}}{2}\pi\right.$$
$$\left.-\sin\frac{(a'-2)\tan\theta+b'-\frac{5}{2}}{2}\pi\right]+\frac{f\tan\theta}{16h\pi(1-\tan\theta)}\left[\sin\frac{-(a'+2)\tan\theta-b'+\frac{5}{2}}{2}\pi\right.$$
$$\left.-\sin\frac{-(a'-2)\tan\theta-b'-\frac{3}{2}}{2}\pi\right]+\frac{f\tan\theta}{16h\pi(1+\tan\theta)}\left[\sin\frac{(a'+2)\tan\theta+b'+\frac{3}{2}}{2}\pi\right.$$
$$\left.-\sin\frac{(a'-2)\tan\theta+b'-\frac{5}{2}}{2}\pi\right]$$

$$\int_{\delta_1}^{\gamma_1} f\,\phi(x-a)\,\phi\left(y-b-\tfrac{1}{2}h\right)dx\cdot\tan\theta$$

$0 \times f_{ib} \neq 0$

No.

Date. 6 · 7 · 2017

$$= \frac{f\tan\theta}{4h} + \frac{f\tan\theta}{8h\pi}\left[\sin\pi - \sin(-\pi)\right] + \frac{f}{8h\pi}\left[\sin\frac{(a'+2)\tan\theta + b' + \frac{1}{2}}{2}\pi - \sin\frac{(a'-2)\tan\theta + b' + \frac{1}{2}}{2}\pi\right] + \frac{f\tan\theta}{16h\pi(1+\tan\theta)}\left[\sin\frac{(a'+2)\tan\theta + b' + \frac{5}{2}}{2}\pi - \sin\frac{(a'-2)\tan\theta + b' - \frac{3}{2}}{2}\pi\right] + \frac{f\tan\theta}{16h\pi(1-\tan\theta)}\left[\sin\frac{-(a'+2)\tan\theta - b' + \frac{3}{2}}{2}\pi - \sin\frac{-(a'-2)\tan\theta - b' - \frac{5}{2}}{2}\pi\right]$$

$$\therefore \frac{f_x(a, b+\frac{1}{2}h) - f_x(a, b-\frac{1}{2}h)}{h}$$

$$= \frac{f\cancel{\tan\theta}}{8h^2\pi}\left[\sin\frac{(a'+2)\tan\theta + b' + \frac{1}{2}}{2}\pi - \sin\frac{(a'+2)\tan\theta + b' - \frac{1}{2}}{2}\pi + \sin\frac{(a'-2)\tan\theta + b' - \frac{1}{2}}{2}\pi - \sin\frac{(a'-2)\tan\theta + b' + \frac{1}{2}}{2}\pi\right] + \frac{f\tan\theta}{16h^2\pi(1+\tan\theta)}\left[\sin\frac{(a'+2)\tan\theta + b' + \frac{5}{2}}{2}\pi + \sin\frac{(a'-2)\tan\theta + b' - \frac{5}{2}}{2}\pi - \sin\frac{(a'-2)\tan\theta + b' - \frac{3}{2}}{2}\pi - \sin\frac{(a'+2)\tan\theta + b' + \frac{3}{2}}{2}\pi\right]$$

$$+ \frac{f\tan\theta}{16h^2\pi(1-\tan\theta)}\left[-\sin\frac{(a'+2)\tan\theta + b' - \frac{3}{2}}{2}\pi - \sin\frac{(a'-2)\tan\theta + b' + \frac{3}{2}}{2}\pi + \sin\frac{(a'-2)\tan\theta + b' + \frac{5}{2}}{2}\pi + \sin\frac{(a'+2)\tan\theta + b' - \frac{5}{2}}{2}\pi\right]$$

No.

Date. 6 · 7 · 2017

Dx $f_{ib} \to 0$

$$= \frac{f}{4h^2\pi}\left[\cos\frac{(a'+2)\tan\theta+b'}{2}\pi\sin\frac{\pi}{4} - \cos\frac{(a'-2)\tan\theta+b'}{2}\pi\sin\frac{\pi}{4}\right]$$

$$+\frac{f\tan\theta}{8h^2\pi(1+\tan\theta)}\left[\sin\frac{(a'+2)\tan\theta+b'}{2}\pi\cos\frac{5}{4}\pi - \sin\frac{(a'-2)\tan\theta+b'}{2}\pi\cos\frac{3}{4}\pi\right]$$

$$+\frac{f\tan\theta}{8h^2\pi(1-\tan\theta)}\left[\sin\frac{(a'-2)\tan\theta+b'}{2}\pi\cos\frac{5}{4}\pi - \sin\frac{(a'+2)\tan\theta+b'}{2}\pi\cos\frac{3}{4}\pi\right]$$

$$= \frac{f}{4h^2\pi}\left[\cos\frac{(a'+2)\tan\theta+b'}{2}\pi\sin\frac{\pi}{4} - \cos\frac{(a'-2)\tan\theta+b'}{2}\pi\sin\frac{\pi}{4}\right]$$

$$+\frac{f\tan\theta}{8h^2\pi(1+\tan\theta)}\left[\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{2\tan\theta+\frac{5}{2}}{2}\pi - \sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{2\tan\theta+\frac{3}{2}}{2}\pi\right]$$

$$+\frac{f\tan\theta}{8h^2\pi(1-\tan\theta)}\left[\sin\frac{a'\tan\theta+b'}{2}\pi\cos\frac{2\tan\theta-\frac{5}{2}}{2}\pi - \sin\frac{a'\tan\theta+b'}{2}\cos\frac{2\tan\theta-\frac{3}{2}}{2}\pi\right]$$

Note that if we take $y = x\tan\theta$, $\theta < 0$, all formulas remain the same except in f_y, $b \to -b$ (change $\theta \to (-\theta) \to -\theta'$ in all equations before integration is carried out to see this)